Coherence and Fluctuations in the Interaction between Moving Atoms and a Quantum Field *

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Abstract

Mesoscopic physics deals with three fundamental issues: quantum coherence, fluctuations and correlations. Here we analyze these issues for atom optics, using a simplified model of an assembly of atoms (or detectors, which are particles with some internal degree of freedom) moving in arbitrary trajectories in a quantum field. Employing the influence functional formalism, we study the self-consistent effect of the field on the atoms, and their mutual interactions via coupling to the field. We derive the coupled Langevin equations for the atom assemblage and analyze the relation of dissipative dynamics of the atoms (detectors) with the correlation and fluctuations of the quantum field. This provides a useful theoretical framework for analysing the coherent properties of atom-field systems.

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1 Mesoscopic Physics in Condensed Matter, Atom Optics, and Cosmology

To practitioners in condensed matter physics, mesoscopia refers to rather specific problems where, for example, the sample size is comparable to the probing scale (nanometers), or the interaction time is comparable to the time of measurement (femtoseconds), or that the electron wavefunction is correlated over the sample thus changing its transport properties fundamentally, or that the fluctuation pattern is reproducible and sample specific. In atom/radiation optics, it is the regime where coherent atom-field interaction, correlations of field, or the effect of boundaries become important. In cosmology, it is the epoch when quantum fluctuations of fields mediate phase transitions, reheat the universe or seed the galaxies. They are described by semiclassical gravity and unified theories from the Planck to the GUT scales. Here we work with a generalized definition of mesoscopia proposed by one of us (see [1], where a general discussion of the conceptual unity among these disciplines can be found), i.e., the quantum / classical, micro / macro interface. It also entails coherent / decoherent, stochastic / deterministic dynamics, and discrete / continuum correspondences. As pointed out in [1], mesoscopia deals with three fundamental issues: quantum coherence, fluctuations and correlations. All mesoscopic processes involve one or more of these aspects.

Many current research directions in early universe cosmology and black hole physics also involve these aspects in a fundamental way. The focus of this talk is however exclusively on atom / radiation optics, which deals with the coherent interaction of atoms and radiation. We will consider the interaction of an atom with a quantum field and examine the coherence, correlation and fluctuations of such a system in a fully non-equilibrium, relativistic field-theoretical treatment [2] [1]. This situation is of basic interest because quantum fields possess zero-point fluctuations which manifest as random forces on an atom. The coherence of the vacuum state also enters in an essential way in the description of atom-field interactions. Here, we shall use a simplified model of an atom, that of a particle with an internal oscillator degree of freedom – call it a detector, moving along an arbitrary trajectory. In fact, to make the correlation aspects even more manifest, we consider an assembly of n such detectors coupled to a quantum field, and study their interaction with the field and their mutual interactions via coupling to the field. This type of problem has been treated before when the atoms (or detectors) are stationary. In [4], for example, there is a discussion of Langevin equations for an arbitrary number of homogenously broadened three-level atoms. Such a treatment, however, holds for atoms fixed in space, and does not consider arbitrary states of motion of the atoms themselves.

On the other hand, nowhere is the role of fluctuations of the vacuum more explicit than in the motion of a uniformly accelerated atom or detector. In the frame moving with such a detector, fluctuations of different modes of the vacuum combine so as to appear as ther-

[1]This treatment is more than necessary for atom optics which deals with slowly moving atoms, and perhaps more befitting for fast moving charged particles in strong fields (plasma physics), but the consistency of backreaction makes such a demand, and it is safer to take the finite temperature, nonrelativistic, far-field, slow motion limits from the final result than as simplifying conditions ab initio. See [3].

mal fluctuations with a temperature proportional to the acceleration (the Unruh effect [5]). Although this effect is considered too small to be directly measurable via mesoscopic experiments at the present time (see [6] for a discussion of numerical estimates on the acceleration for a measurable Unruh effect), it does lead to shifts in the energy levels of two-level atoms which are qualitatively of the same form as the Lamb shift, and can therefore be ascribed to vacuum fluctuations. Atomic energy shifts resulting from accelerated motion, and their origin in vacuum fluctuations and radiation reaction have been investigated [7]. It has also been recently shown [8] that acceleration induces new Raman-like transitions in multi-level atoms.

Experiments will soon reach the stage where such delicate field-theoretical and statistical mechanical attributes of an atom-field system, and their sensitivity to different states of motion, will become important and measurable. Thus it is instructive to examine the effect of introducing two types of theoretical "probes" into the coherence and fluctuation properties of a quantum state. One type of probe involves introducing a large number of atoms and allowing the quantized field to mediate interactions between them, thus setting up correlations in such a many-body system. The other type of probe is to ascribe arbitrary states of motion to the atoms themselves, thus amplifying vacuum fluctuations. It is highly likely that consideration of such problems will lead to new ways of understanding phenomena associated with vacuum fluctuations and quantum coherence.

We treat the simplest version of such a problem in the next section, details of which can be found in [2]. We find that the dynamics of a many detector system can be described by a set of coupled Langevin equations, and that the noise sources in these Langevin equations are governed by vacuum fluctuations of the field at the site of the detector and by quantum correlations of the vacuum between different detector sites. Also, as we have shown in [2], these equations lead to the following conclusions:

a) Vacuum fluctuations are related linearly (albeit nonlocally in general) to the dissipative or radiation reaction self-force on the atom by a fluctuation dissipation relation which holds for atoms on arbitrary trajectories (not just stationary trajectories, as is usually assumed in the literature – see [7] for example).

b) The form of the dissipative radiation reaction force is independent of the trajectory in our model (in [7] the same statement is proved independently for stationary trajectories using a realistic QED model, thus lending support to its general validity).

c) Finally, correlations of the vacuum between different detector sites are related linearly to the radiation mediated between them for trajectories without horizons (all realistic particle trajectories fall in this class). This leads us to generalizations of the fluctuation- dissipation relation to "correlation-propagation" relations, explicitly displayed in [2].

All of the above three statements, if shown to hold for realistic atom optics, are experimentally testable in principle and should be essential components of a non-equilibrium description of mesoscopic atom optics systems. We will now go on to outline the relevant details of our model.

2 Coherent Atom Optics: Moving Atoms in Scalar Electrodynamics Model

We use a simplified model which describes the coupling of some charged particles (we shall interchangeably call them atoms or detectors) with some internal degrees of freedom to a massless scalar field. Although realistic atoms are electrically neutral, this type of interaction is similar to the interaction of the atomic dipole moment with a electromagnetic field. To find out the effect of the field on the detectors moving along arbitrary trajectories, we derive the influence functional, and from it the coupled Langevin equations of motion for the system of N detectors. The effect of correlation, dissipation and fluctuations can be extracted. We assume that the field and the system of detectors are initially decoupled from each other, and that the field is initially in the Minkowski vacuum state. We consider a 1+1 dimensional model, even though the formalism can be simply extended to higher dimensions, and to different choices of initial state for the field, for example a finite temperature density matrix. The path integral influence functional formalism we use is somewhat different from the operator formalisms usually employed in atom optics. It allows one to characterize fluctuation and dissipation arising from the field variables which are integrated out, in a natural way. It is also a self-consistent, non-perturbative treatment (the coupling constant between atom and field need not be treated as a small parameter when the Lagrangian is quadratic in all variables) and takes into account the full backreaction of the field on the atoms.

Consider N detectors or atoms $i = 1, ..N$ in $1 + 1$ dimensions with internal coordinates $Q_i(\tau_i)$ modeled as oscillator degrees of freedom, and moving on trajectories $(x_i(\tau_i), t_i(\tau_i))$, τ_i being a parameter along the trajectory of detector i. In the following analysis, we do not need to assume that τ_i is the proper time, although this is, in most cases, a convenient choice. However, we will assume hereafter that the trajectories $(t_i(\tau_i), x_i(\tau_i))$ are smooth and that the parameters τ_i are chosen such that $t_i(\tau_i)$ is a strictly increasing function of τ_i.

The detectors are coupled to a massless scalar field $\phi(x, t)$ via the interaction action

$$S_{int} = \sum_i e_i \int_{-\infty}^{t_i^{-1}(T)} d\tau_i s_i(\tau_i) \frac{dQ_i}{d\tau_i} \phi(x_i(\tau_i), t_i(\tau_i)). \tag{2.1}$$

Here, T is a global Minkowski time coordinate which defines a spacelike hypersurface, e_i denotes the coupling constant of detector i to the field, $s_i(\tau_i)$ is a switching function for detector i (typically a step function), and t_i^{-1} is the inverse function of t_i. $t_i^{-1}(T)$ is therefore the value of τ_i at the point of intersection of the spacelike hypersurface defined by T with the trajectory of detector i. Note that the strictly increasing property of $t_i(\tau_i)$ implies that the inverse, if it exists, is unique.

The action of the system of detectors is

$$S_{osc} = \frac{1}{2} \sum_i \int_{-\infty}^{t_i^{-1}(T)} d\tau_i [(\partial_{\tau_i} Q_i)^2 - \Omega_i^2 Q_i^2]. \tag{2.2}$$

The scalar field action is given by

$$S_{field} = \frac{1}{2}\int_{-\infty}^{T} dt \int dx[(\partial_t\phi)^2 - (\partial_x\phi)^2] \tag{2.3}$$

and the complete action

$$S = S_{field} + S_{osc} + S_{int}. \tag{2.4}$$

Expanding the field in normal modes,

$$\phi(x,t) = \sqrt{\frac{2}{L}}\sum_k{}' [q_k^+(t)\cos kx + q_k^-(t)\sin kx] \tag{2.5}$$

where $\sum_k'$ denotes that the summation is restricted to the upper half k space, $k > 0$. Then the action for the scalar field is given by $(\sigma = +, -)$

$$S_{field} = \frac{1}{2}\sum_{k,\sigma}{}' [(\dot{q}_k^\sigma)^2 - \omega_k^2 q_k^2] \tag{2.6}$$

and the interaction action is

$$\begin{aligned} S_{int} &= \sum_i e_i\sqrt{\frac{2}{L}}\int_{-\infty}^{t_i^{-1}(T)} d\tau_i \frac{dQ_i}{d\tau_i} \times \\ &\quad \sum_k{}' [q_k^+(t_i(\tau_i))\cos kx_i(\tau_i) + q_k^-(t_i(\tau_i))\sin kx_i(\tau_i)]s_i(\tau_i) \\ &= \sum_i e_i\sqrt{\frac{2}{L}}\int_{-\infty}^{\infty} dt \int_{-\infty}^{t_i^{-1}(T)} d\tau_i \delta(t - t_i(\tau_i))\frac{dQ_i}{d\tau_i} \times \\ &\quad \sum_k{}' [q_k^+(t)\cos kx_i(\tau_i) + q_k^-(t)\sin kx_i(\tau_i)]s_i(\tau_i). \end{aligned} \tag{2.7}$$

We have $t_i(\tau_i) < T$, which follows from $\tau_i < t_i^{-1}(T)$ and the property that $t_i(\tau_i)$ is a strictly increasing function. Hence we may replace the upper limit of the dt integration by T. This replacement leads to the expression:

$$S_{int} = -\sum_{k,\sigma}{}' \int_{-\infty}^{T} dt J_k^\sigma(t) q_k^\sigma(t) \tag{2.8}$$

where

$$J_k^\sigma(t) = -\sum_i e_i\sqrt{\frac{2}{L}}\int_{-\infty}^{t_i^{-1}(T)} d\tau_i \delta(t - t_i(\tau_i))\frac{dQ_i}{d\tau_i} u_k^\sigma(\tau_i)s_i(\tau_i) \tag{2.9}$$

and

$$u_k^+(\tau_i) = \cos kx_i(\tau_i); \quad u_k^-(\tau_i) = \sin kx_i(\tau_i). \tag{2.10}$$

The action $S_{field}+S_{int}$ therefore describes a system of decoupled harmonic oscillators each driven by separate source terms. The zero temperature influence functional (corresponding to the initial state of the field being the Minkowski vacuum state) for this system has the form [9]:

$$\mathcal{F}[J,J'] = \exp -\frac{1}{\hbar}\sum_{k,\sigma}{}' \int_{-\infty}^{T} ds \int_{-\infty}^{s} ds' [J_k^\sigma(s) - J_k'^\sigma(s)][\zeta_k(s,s')J_k^\sigma(s') - \zeta_k^*(s,s')J_k'^\sigma(s')] \tag{2.11}$$

where

$$\zeta_k \equiv \nu_k + i\mu_k = \frac{1}{2\omega_k} e^{-i\omega_k(s-s')}. \tag{2.12}$$

If the field is initially in a thermal state, the influence functional has the same form as above, and the quantity ζ_k becomes

$$\zeta_k = \frac{1}{2\omega_k}\left[\coth\left(\frac{1}{2}\beta\omega_k\hbar\right)\cos\omega_k(s-s') - i\sin\omega_k(s-s')\right], \tag{2.13}$$

β being the inverse temperature. We shall restrict our attention to the zero temperature case.

Substituting for the J_k^σ's in the influence functional, and carrying out the δ-function integrations, one obtains

$$\begin{aligned}\mathcal{F}[\{Q\};\{Q'\}] \quad = \quad & \exp -\frac{1}{\hbar}\left\{\sum_{i,j=1}^{N}\int_{-\infty}^{t_i^{-1}(T)} d\tau_i s_i(\tau_i)\int_{-\infty}^{t_j^{-1}(t_i(\tau_i))} d\tau_j' s_j(\tau_j')\left[\frac{dQ_i}{d\tau_i} - \frac{dQ_i'}{d\tau_i}\right]\times\right.\\ & \left.\left[Z_{ij}(\tau_i,\tau_j')\frac{dQ_j}{d\tau_j'} - Z_{ij}^*(\tau_i,\tau_j')\frac{dQ_j'}{d\tau_j'}\right]\right\}\end{aligned} \tag{2.14}$$

where

$$Z_{ij}(\tau_i,\tau_j') = \frac{2}{L}e_i e_j \sum_{k,\sigma}{}' \zeta_k(t_i(\tau_i),t_j(\tau_j'))u_k^\sigma(\tau_i)u_k^\sigma(\tau_j'). \tag{2.15}$$

In the above, the continuum limit in the mode sum is recovered through the replacement $\sum_k' \to \frac{L}{2\pi}\int_0^\infty dk$. We then obtain, after substituting for u_k^σ and ζ_k,

$$Z_{ij}(\tau_i,\tau_j') = \frac{e_i e_j}{2\pi}\int_0^\infty \frac{dk}{k} e^{-ik(t_i(\tau_i)-t_j(\tau_j'))}\cos k(x_i(\tau_i) - x_j(\tau_j')). \tag{2.16}$$

In this form, Z_{ij} is proportional to the two point function of the free scalar field in the Minkowski vacuum, evaluated for the two points lying on trajectories i and j of the detector system. It obeys the symmetry relation

$$Z_{ij}(\tau_i,\tau_j') = Z_{ji}^*(\tau_j',\tau_i) \tag{2.17}$$

Corresponding to (2.12), we may also split Z_{ij} into its real and imaginary parts. Thus we define

$$Z_{ij}(\tau_i,\tau_j') = \tilde{\nu}_{ij}(\tau_i,\tau_j') + i\tilde{\mu}_{ij}(\tau_i,\tau_j') \tag{2.18}$$

where

$$\begin{aligned}
\tilde{\nu}_{ij}(\tau_i,\tau_j') &= \frac{e_ie_j}{2\pi}\int_0^\infty \frac{dk}{k}\cos k(t_i(\tau_i)-t_j(\tau_j'))\cos k(x_i(\tau_i)-x_j(\tau_j')) \\
\tilde{\mu}_{ij}(\tau_i,\tau_j') &= -\frac{e_ie_j}{2\pi}\int_0^\infty \frac{dk}{k}\sin k(t_i(\tau_i)-t_j(\tau_j'))\cos k(x_i(\tau_i)-x_j(\tau_j')).
\end{aligned} \tag{2.19}$$

$\tilde{\nu}$ and $\tilde{\mu}$ are proportional to the anticommutator and the commutator of the field in the Minkowski vacuum, respectively.

The quantities Z_{ij} are also conveniently expressed in terms of advanced and retarded null coordinates $v_i(\tau_i) = t_i(\tau_i) + x_i(\tau_i)$ and $u_i(\tau_i) = t_i(\tau_i) - x_i(\tau_i)$, as

$$Z_{ij}(\tau_i,\tau_j') = Z^a_{ij}(\tau_i,\tau_j') + Z^r_{ij}(\tau_i,\tau_j') \tag{2.20}$$

where

$$\begin{aligned}
Z^a_{ij}(\tau_i,\tau_j') &= \frac{e_ie_j}{4\pi}\int_0^\infty \frac{dk}{k}e^{-ik(v_i(\tau_i)-v_j(\tau_j'))} \\
Z^r_{ij}(\tau_i,\tau_j') &= \frac{e_ie_j}{4\pi}\int_0^\infty \frac{dk}{k}e^{-ik(u_i(\tau_i)-u_j(\tau_j'))}
\end{aligned} \tag{2.21}$$

and the superscripts a and r denote advanced and retarded respectively.[2] Similar decompositions for $\tilde{\nu}_{ij}$ and $\tilde{\mu}_{ij}$ thus follow.

The influence functional, together with the free action for the detector system, can be employed to obtain the propagator for the reduced density matrix of the system of detectors. This propogator will contain complete information about the dynamics of the detectors. However, we shall take the alternative route of deriving Langevin equations for the detector system in order to describe its dynamics.

3 Stochastic Dynamics of Atom-Field Interactions: Dissipation, Fluctuations and Correlations

So far, we have shown how to coarse-grain the field degrees of freedom and incorporate their effect on the detectors which manifest as long-range interactions between the various detectors. We now derive the effective stochastic equations of motion for the N-detector system. These equations should be considered strictly useful only after the system of detectors has effectively decohered and a consistent classical description then becomes valid. Only in this regime can stochastic variables replace the true quantum variables to good approximation. Physically, one is usually interested in looking at the system long after its transient behavior is damped out (i.e. after the relaxation time), and in this regime the effective stochastic

[2] The terminology 'advanced' and 'retarded' refers to the null coordinates. Equivalently, they can be called 'left-moving' and 'right-moving', respectively, when the sense of motion refers to the future direction in time. This terminology is used in wave theory and string theory.

description is quite sound[3]. The reason we prefer such a description is that it allows us to characterize quantum noise from the field as a bonafide stochastic variable, thus making possible a natural probabilistic interpretation for the quantum fluctuation-dissipation relation.

Going back to the form (2.11) for the influence functional, we define the centre of mass and relative variables

$$\begin{aligned} J_k^{+\sigma}(s) &= (J_k^\sigma(s) + J_k^{'\sigma}(s))/2 \\ J_k^{-\sigma}(s) &= J_k^\sigma(s) - J_k^{'\sigma}(s). \end{aligned} \tag{3.1}$$

Correspondingly, we also find it convenient to define

$$\begin{aligned} Q_i^+(\tau_i) &= (Q_i(\tau_i) + Q_i'(\tau_i))/2 \\ Q_i^-(\tau_i) &= Q_i(\tau_i) - Q_i'(\tau_i). \end{aligned} \tag{3.2}$$

Then Equation (2.11) yields

$$\mid \mathcal{F}[J, J'] \mid \; = \; \exp\{-\frac{1}{\hbar}\sum_{k,\sigma}{}' \int_{-\infty}^{T} ds \int_{-\infty}^{s} ds' J_k^{-\sigma}(s)\nu_k(s,s')J_k^{-\sigma}(s')\} \tag{3.3}$$

$$= \int \Pi'_{k,\sigma}(\mathcal{D}\xi_k^\sigma P[\xi_k^\sigma]) \exp -\frac{i}{\hbar}\sum_{k,\sigma}{}' \int_{-\infty}^{T} ds J_k^{-\sigma}(s)\xi_k^\sigma(s). \tag{3.4}$$

$\mid \mathcal{F} \mid$ is the absolute value of $\mathcal{F}$, containing the kernel ν_k. The phase of $\mathcal{F}$ contains the kernel μ_k. In the second equality, we have used a functional gaussian integral identity, $P[\xi_k^\sigma]$ being the positive definite measure

$$P[\xi_k^\sigma] = N \exp\{-\frac{1}{2\hbar}\int_{-\infty}^{T} ds \int_{-\infty}^{T} ds' \xi_k^\sigma(s)\nu_k^{-1}(s,s')\xi_k^\sigma(s')\} \tag{3.5}$$

normalized to unity. It can therefore be interpreted as a probability distribution over the function space ξ_k^σ.

The influence functional can thus be expressed as

$$\begin{aligned} \mathcal{F}[\{Q\},\{Q'\}] &= \; < \exp\left\{-\frac{i}{\hbar}\sum_{k,\sigma}{}' \int_{-\infty}^{T} ds J_k^{-\sigma}(s)\left[\xi_k^\sigma(s) + 2\int_{-\infty}^{s} ds' \mu_k(s,s')J_k^{+\sigma}(s')\right]\right\} > \\ &\equiv \; < \exp\frac{i}{\hbar}S_{inf} > \end{aligned} \tag{3.6}$$

where $< \;\; >$ denotes expectation value with respect to the joint distribution $\Pi'_{k,\sigma}P[\xi_k^\sigma]$. S_{inf} will be called the stochastic influence action. We find

$$\begin{aligned} < \xi_k^\sigma(s) > &= 0, \\ < \{\xi_k^\sigma(s), \xi_{k'}^{\sigma'}(s')\} > &= \hbar\delta_{kk'}\delta_{\sigma\sigma'}\nu_k(s,s') \end{aligned} \tag{3.7}$$

[3]The stochastic description should be sound even over time scales much shorter than the relaxation time scale, but longer than the decoherence time scale. However, the Langevin equations derived here cannot be used to study the process of decoherence itself.

where { , } denotes the anticommutator.

Substituting for $J_k^{-\sigma}$ and $J_k^{+\sigma}$ in terms of the detector degrees of freedom $\{Q_i\}$, the stochastic influence action S_{inf} is obtained as

$$S_{inf} = -\sum_{i=1}^{N}\int_{-\infty}^{\tau_i(T)} d\tau_i \frac{dQ_i^-}{d\tau_i} s_i(\tau_i)\left[\eta_i(\tau_i) + 2\sum_{j=1}^{N}\int_{-\infty}^{\tau_j(t_i(\tau_i))} d\tau_j' \frac{dQ_j^+}{d\tau_j'} s_j(\tau_j')\tilde{\mu}_{ij}(\tau_i,\tau_j')\right] \tag{3.8}$$

with

$$\eta_i(\tau_i) = e_i {\sum_{k,\sigma}}' \sqrt{\frac{2}{L}} u_k^\sigma(\tau_i)\xi_k^\sigma(t_i(\tau_i)). \tag{3.9}$$

From Equation (3.8) we see that the quantities $\tilde{\mu}_{ij}$, $i \neq j$ mediate long-range interactions between the various detectors and the quantities $\tilde{\mu}_{ii}$ describe self-interaction of each detector due to its interaction with the field. This self-interaction typically manifests itself as a dissipative (or radiation reaction) force in the dynamics of the detectors. We will, therefore, refer to $\tilde{\mu}_{ij}$, $i \neq j$ as a "propagation kernel", and $\tilde{\mu}_{ii}$ as a "dissipation kernel".

We now turn to the interpretation of the quantities η_i. They appear as source terms in the effective action of the detector system. Also, being linear combinations of the quantities ξ_k^σ, they are stochastic in nature. Indeed, from Equations (3.7) and (3.9) we can obtain

$$\begin{aligned} < \eta_i(\tau_i) > &= 0, \\ < \{\eta_i(\tau_i), \eta_j(\tau_j')\} > &= e_i e_j {\sum_{k,\sigma}}' {\sum_{k',\sigma'}}' u_k^\sigma(\tau_i) u_{k'}^{\sigma'}(\tau_j')(\frac{2}{L}) < \xi_k^\sigma(t_i(\tau_i))\xi_{k'}^{\sigma'}(t_j(\tau_j')) > \\ &= \hbar\tilde{\nu}_{ij}(\tau_i,\tau_j'). \end{aligned} \tag{3.10}$$

Thus $\tilde{\nu}_{ij}$ appears as a correlator of the stochastic forces η_i and η_j. Along a fixed trajectory, this correlation manifests as noise in the detector dynamics. Hence we call $\tilde{\nu}_{ii}$ a "noise kernel" and $\tilde{\nu}_{ij}$, $i \neq j$, a "correlation kernel".[4]

The full stochastic effective action for the N-detector system is given by

$$S_{eff} = S_{osc} + S_{inf}. \tag{3.11}$$

We may now express this in terms of the variables Q_i^+ and Q_i^- defined earlier. Thus we obtain

$$\begin{aligned} S_{eff} &= \sum_{i=1}^{N}\int_{-\infty}^{\tau_i(T)} d\tau_i[\dot{Q}_i^-\dot{Q}_i^+ - \Omega_i^2 Q_i^- Q_i^+ - \dot{Q}_i^- s_i(\tau_i)\eta_i(\tau_i) \\ &\quad -2\dot{Q}_i^- s_i(\tau_i)\sum_{j=1}^{N}\int_{-\infty}^{\tau_j(t_i(\tau_i))} d\tau_j' \dot{Q}_{j'}^+ s_j(\tau_j')\tilde{\mu}_{ij}(\tau_i,\tau_j')] \end{aligned} \tag{3.12}$$

[4]The distinction between noise and correlation is unnecessary from the point of view of the field. 'Noise', as used here, also represents free field correlations for points on a single trajectory. However, from the point of view of each detector, these two quantities play a different role. Hence the choice of terminology.

where $\dot{f}_i \equiv \frac{df_i}{d\tau_i}$, $\dot{f}_{j'} \equiv \frac{df_j}{d\tau'_j}$.

Extremizing the effective action with respect to Q_i^- and setting $Q_i = Q'_i$ at the end [9], we obtain a set of coupled equations of motion, the Langevin equations, for the system of detectors:

$$\frac{d^2Q_i}{d\tau_i^2} - 2\sum_{j=1}^{N}\int_{-\infty}^{\tau_j(t_i(\tau_i))} d\tau'_j s_j(\tau'_j)\frac{d}{d\tau_i}(s_i(\tau_i)\tilde{\mu}_{ij}(\tau_i,\tau'_j))\frac{dQ_j}{d\tau'_j} + \Omega_i^2 Q_i = \frac{d}{d\tau_i}(s_i(\tau_i)\eta_i(\tau_i)). \quad (3.13)$$

Due to the back-reaction of each detector on the field, and consequently on other detectors, the effective dynamics of the detector system is highly non-trivial and, as such, can be solved in closed form only for simple trajectories or under simplifying assumptions such as ignoring the back-reaction of certain detectors on the field. For instance, if we choose to ignore the back-reaction of detector i on the field, this can be effected by setting $\tilde{\mu}_{ji} = 0$, for all j, including $j = i$,while at the same time keeping $\tilde{\mu}_{ij} \neq 0$ for $j \neq i$. The particular case $\tilde{\mu}_{ii} = 0$ amounts to ignoring the radiation reaction of detector i. This is necessary because the radiation reaction effect arises due to a modification of the field in the vicinity of the detector as a consequence of the back-reaction of the detector on the field[5].

Our formal treatment of the detector-field system is exact in that it includes the full back-reaction of the detectors on the field, which is manifested in the coupled Langevin equations of the various detectors. These coupled equations of motion give rise to a sort of "dynamical correlation" between the various detectors. Non-dynamical correlations also occur because of the intrinsic correlations in the state of the field (Minkowski vacuum). These correlations are purely quantum-mechanical in origin, and they are reflected in the correlators of the stochastic forces, $\tilde{\nu}_{ij}$. Correlations between stochastic forces on different detectors induce correlations between the coordinates Q_i of different detectors.

As commented earlier, and shown explicitly in [2], our exact treatment makes it possible to demonstrate the existence of generalized fluctuation-dissipation relations relating the fluctuations of the stochastic forces on the detectors to the dissipative forces. We also discovered a related set of correlation-propagation relations between the correlations of stochastic forces on different detectors and the retarded and advanced parts of the radiation mediated by them. These relations are of categorical nature and hence of fundamental significance in the desciption of moving atoms interacting with a field.

[5]Of course, it is in general inconsistent to ignore the back-reaction of a detector, as it leads to a direct violation of the symmetry (2.17). As is well-known, it also leads to unphysical predictions. For example, in the treatment of an atom on an inertial trajectory, coupled to a quantum field, balance of vacuum fluctuations and radiation reaction is necessary to ensure the stability of the ground state. As explained above, ignoring back-reaction implies ignoring the radiation reaction force. Such a treatment would render the ground state unstable. However, in certain cases, the quantities $\tilde{\mu}_{ji}$ may not contribute to the dynamics of detector j. This occurs, for example, when the trajectory of one detector is always outside the causal future of the other one. Hence there is no retarded effect of one of the detectors on the other.

4 Applications

As applications of these Langevin equations, we have considered in [2] four examples of increasing complexity: a) a single detector in the Minkowski vacuum moving on an inertial trajectory, b) a single detector on a uniformly accelerated trajectory, c) two detectors on inertial trajectories, and d) one detector on a uniformly accelerated trajectory and another one on an arbitrary trajectory, functioning as a probe. In all cases, we can solve exactly for the detector coordinates, at least in the late time limit (this limit is actually realized at any finite time when the two detectors have been switched on forever, and corresponds to the neglect of transients in the solutions for the detector coordinates).

Case a) describes the well-known physical effects in quantum field theory of the dressing of a particle by the field. Case b) describes the Unruh effect [5] where thermal radiation from the excitation of quantum noise is experienced by a uniformly accelerated particle. In c) we introduce the notions of "self" and "mutual" impedance which govern the response of either detector. The effect of the back-reaction of each detector on the field and consequently on the other detector is to introduce the so-called mutual impedance in the detector response as well as to modify the self-impedance of each detector from its value in the absence of the other one. In d) we switch on the probe after it intersects the future horizon of the uniformly accelerated detector, so that it cannot causally influence the uniformly accelerated one. Because of this, the dissipative features of this problem are relatively trivial. The response of the probe has contribution mainly from field correlations across the horizon. On the other hand, the noise due to field fluctuations and the field correlations between the two trajectories play a dominant role. This correlation can be expressed in terms of noise via a correlation-propagation relations which are appropriate extensions of a generalized fluctuation-dissipation relation directly relating field fluctuations to dissipative properties of the detectors.

Here we have focussed on the correlation and fluctuation aspects of the atom-field system. In more practical problems in atom optics, one can use a suitable generalization of the Langevin equations derived here for the description of dissipative atom motion (with radiative-reaction). The noise correlators describe the stochastic source from the vacuum fluctuations of the field as they appear to moving atoms, and the fluctuation-dissipation and correlation-propagation relations relate the dissipative effect of the atom to the correlations and fluctuations of the field. Similar methods are now applied to two-level atom systems [10] and detector motion with non-prescribed trajectories (determined by backreaction of the field) [3]. It is hoped that studies such as these will bring new insights into the coherence and fluctuation properties of the quantum vacuum state, and further confirm the relevance of these properties to mesoscopic physics via system-field interactions.

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References

[1] B. L. Hu, "Semiclassical Gravity and Mesoscopic Physics" in *Quantum Classical Correspondence – Proceedings of the Fourth Drexel symposium on Quantum Nonintegrability*, Sept. 8-11, 1994, Philadelphia. Edited by D. H. Feng and B. L. Hu (International Press, Boston, 1997).

[2] A. Raval, B. L. Hu and J. R. Anglin, Phys. Rev. D53, 7003 (1996).

[3] P. Johnson and B. L. Hu, "Relativistic Particle Detector Motion in a Quantum Field: Radiation Reaction and Vacuum Fluctuations".

[4] *Quantum Statistical Properties of Radiation*, by William H. Louisell, John Wiley and Sons, N.Y. (1973).

[5] W. G. Unruh, Phys. Rev. **D14**, 3251 (1976); Phys. Rev. **D 46**, 3271 (1992). P. G. Grove, Class. Quan. Grav. **3**, 801 (1986) D. J. Raine, D. W. Sciama, and P. G. Grove, Proc. Roy. Soc. Lond. **A435**, 205 (1991). S. Massar, R. Parentani and R. Brout, Class. Quantum Grav. 10, 385 (1993).

[6] H. C. Rosu, Int. J. Mod. Phys. D3, 545 (1994).

[7] J. Audretsch and R. Muller, Phys. Rev. D 49, 6566 (1994); A 50, 1755 (1994); A 52, 629 (1995).

[8] Karl-Peter Marzlin and Jurgen Audretsch, preprint gr-qc/9707058.

[9] B. L. Hu, J. P. Paz and Y. Zhang, Phys. Rev. D45, 2843 (1992); D47, 1576 (1993).

[10] C. Anastopoulos and B. L. Hu, "Atom-Field Interaction: Nonequilibrium Dynamics via the Influence Functional".